FOURIER-MOTZKIN

Fourier-Motzkin eliminates all quantifiers (keeping logical equivalence over the structure of the reals) until we get a ground constraints that evaluates to true or false

Advantages:

* easy to explain
* in some sense it is powerful

Disadvantages

* not the best procedure in sense of implementation

# General Instruction

## Remove negative literals

t ≠ u becomes t < u v t > u

## Rewrite positive literals by isolating x as a first member

In this way you get only literals in the form

x = t

x > t

x < t

(Where t is a linear polynomial without x )

## Remove x if it has a “definition”

x = t ^ A becomes A(t/x)

## Collect the terms

There can be three types of formulas

(α) x < t1 ^ x < t2 ^ … ^ x < tn

(β) t1 < x ^ t2 < x ^ … ^ tn < x

(γ) formula where x does not occur (like t1 < t2 or 0 = 0 if x occur always )

# Example 1

3y + x < 0 ^ x + 6 - 2y > 0 ^ y > 0 ^ x < 1

First we need to isolate x, that is

x < -3y ^ x > 2y - 6 ^ y > 0 ^ x < 1

Eliminate the x by substitution

2y - 6 < -3y ^ 2y - 6 < 1 ^ y > 0

Than we isolate the y

y < 6/5 ^ y < 7/2 ^ y > 0

Eliminate the y

0 < 6/5 ^ 0 < 7/2

Since both are SAT, it is SAT

# Example 2

x = 3y + 2 ^ x < 2 ^ 4y > x

Rewrite with the x in front

x = 3y + 2 ^ x < 2 ^ x < 4y

Elimite x by substitution

3y + 2 < 2 ^ 3y + 2 < 4y

Isolate the y

y < 0 ^ 2 < y

Eliminate the y by substitution

0 > 2

UNSAT

# Example 3

x < y ^ x > 2y ^ y < 7

Eliminate the x by substitution

2y < y ^ y < 7

Isolate y

y < 0 ^ y < 7

So 0 < y < 7

SAT